PHYS 301
Electricity and Magnetism

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Today!

- Electric potential
- Conductors
- Capacitors
- Magnetic fields

Fundamental Equations of Electrostatics

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_o \qquad \vec{\nabla}\times\vec{E} = 0$$

permits $\vec{E} = -\vec{\nabla}V$

- In terms of potential:

$$\vec{\nabla}\cdot(-\vec{\nabla}V) = -\nabla^2 V = \rho/\varepsilon_o$$

$$\nabla^2 V = -\rho/\varepsilon_o$$

Poisson's equation

if $\rho = 0$ →

$$\nabla^2 V = 0$$

LaPlace's equation

Electric Potential

ELECTROSTATICS

- The workhorse of electric potential looks a lot like its electric field counter part:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\frac{q}{\boldsymbol{\mathscr{r}}}$$

point charge

continuous charge distributions

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int\frac{\rho(\vec{r}\,')}{|\vec{\boldsymbol{\mathscr{r}}}|}d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int\frac{\sigma(\vec{r}\,')}{|\vec{\boldsymbol{\mathscr{r}}}|}dA'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int\frac{\lambda(\vec{r}\,')}{|\vec{\boldsymbol{\mathscr{r}}}|}dl'$$

relative to what?

infinity!

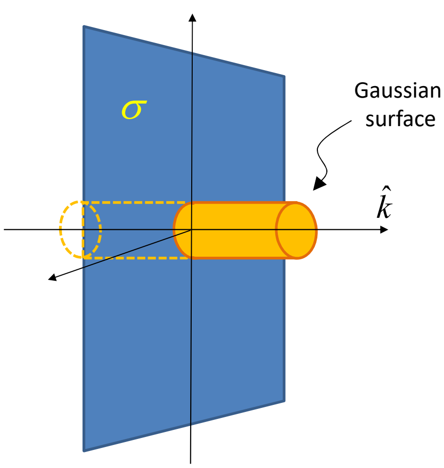
Conductors

ELECTROSTATICS

- **Ideal conductors:** Material with unlimited supply of completely free electrons! [fiction]
- **Properties:**
  - $\vec{E} = 0$ inside a conductor.
  - $\rho = 0$ inside a conductor.
  - Any net charge resides on the outer surface
  - V is constant throughout a conductor.
  - $\vec{E}$ is normal to the surface just outside a conductor.
- **Charging by induction:** Electric fields can induce charge separation in a conductor

Gauss' Law:

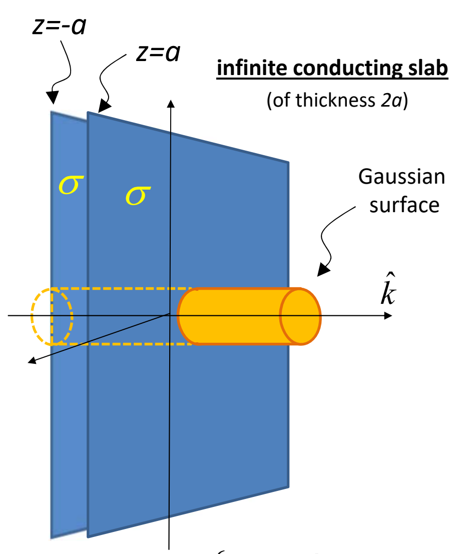
infinite sheet



$$\vec{E} = \begin{cases} +\sigma/2\varepsilon_o \hat{k}, \ for\ z>0 \\ -\sigma/2\varepsilon_o \hat{k}, \ for\ z<0 \end{cases}$$

infinite conducting slab

(of thickness *2a*)



$$\vec{E} = \begin{cases} +\sigma/\varepsilon_o \hat{k}, \ for\ z>a \\ -\sigma/\varepsilon_o \hat{k}, \ for\ z<-a \end{cases}$$

ELECTROSTATIC BOUNDARY CONDITIONS

- THE NORMAL COMPONENT OF $\vec{E}$ IS DISCONTINUOUS BY AN AMOUNT σ/ε_o AT *ANY* BOUNDARY:

$$\vec{E}_{above} - \vec{E}_{below}\Big|_{boundary} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

"ABOVE" AND "BELOW THE BOUNDARY!

WHERE $\hat{n} =$ unit normal vector to the surface

➡ THE PARALLEL COMPONENT OF $\vec{E}$ <u>IS</u> CONTINUOUS.

- THE ELECTRIC POTENTIAL IS CONTINUOUS ACROSS ANY BOUNDARY:

$$V_{above}\Big|_{boundary} = V_{above}\Big|_{boundary}$$

➡ SINCE $\vec{E} = -\vec{\nabla} V$, $\vec{\nabla} V_{above} - \vec{\nabla} V_{below} = -\frac{\sigma}{\varepsilon_o}\hat{n}$

OR

$$\vec{\nabla} V_{above} \cdot \hat{n} - \vec{\nabla} V_{below} \cdot \hat{n}\Big|_{boundary} = \frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n}\Big|_{boundary} = -\frac{\sigma}{\varepsilon_o}$$

"NORMAL DERIVATIVE OF V"

Conductors

ELECTROSTATIC BOUNDARY CONDITIONS

- The electric field immediately <u>outside</u> the surface of a conductor is

$$\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

- Similarly, the potential is then expressed as

$$\frac{\partial V}{\partial n} = -\frac{\sigma}{\varepsilon_o}$$

Solutions to LaPlace's Equation $\nabla^2 V = 0$

- Called *harmonic functions*
- No points of stable equilibrium (*electrostatic*)
- No local minima or maxima

(*membrane stretched over frame*)

in 2 D

- Thus, the value of V(r) at point P is the average value of V(r) over a spherical surface of radius R centered at P

$$V(\vec{r}) = \frac{1}{4\pi R^2} \oint_{sphere} V \, dA'$$

- Suggests computation method:

relaxation method

Uniqueness Theorems $\nabla^2 V = 0$

- The solution to LaPlace's equation in some region of space is **uniquely determined** if the value of V(r) is specified on the **boundaries** of the region.
  - i.e., V(r) is uniquely determined if ρ(r) and V(r) on the boundaries are given.
- In a region containing conductors and a specified ρ(r), the electric field is **uniquely determined** if the total charge on each conductor is given.

ELECTROSTATICS

Method of Images

$$\nabla^2 V = 0$$

- Solution? Whatever works!
- The classic problem:

Find V in region above the plane!